

Images of Chaos – computed Fractals

It is well known that the equation $x^2 = -1$ has no solution within the real numbers, because real squares cannot be negative. In order to solve such equations as well the number $i = \sqrt{-1}$ is introduced first. You create a new axis of numbers, the imaginary numbers that are perpendicular to the straight line of the real numbers. On this axis, the number i is used as the unit.

With these numbers you can calculate similar to the real numbers, you only have to introduce some additional rules, which are compatible with the calculation rules of the real numbers. Rectangular sections from this area are wonderfully suitable as picture areas.

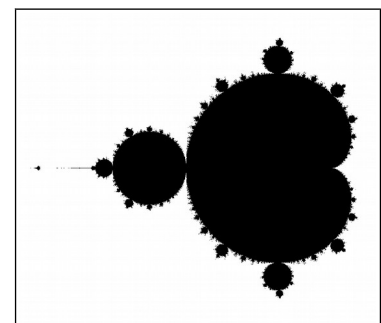
Chaotic structures are created by repeated application of non-linear operations.

A sequence of complex numbers is defined by an iteration process, whereby the next number z_{n+1} is determined by a calculation rule from the previous number z_n . The start value z_0 is fixed at the beginning. Usually this is the zero point. It is interesting to follow the path of the sequence at the plane of complex numbers. Either the points remain within a circle around the zero point, or they move further and further out without a border.

A nonlinear iteration process is defined by $z_{n+1} = z_n^2 + c$, for example. As initial value $z_0=0$ is set. A point c is selected from the complex number plane, and for this c the sequence z_n is calculated bit by bit. For $c=0$, the sequence always remains at $z_n=0$, for $c=1$, however, the sequence remains on the real number line, but with **1, 2, 5, 26, 677...** it quickly grows beyond all limits. For some starting points c the elements of the sequence remain within a fixed circle around the zero point, for others not. Depending on this behavior of the sequence, point c on the plane can be colored black or white. If you calculate sequences for each point c of the plane, you get (of course limited to the available pixels of our image section) the following representation:

The now white colored area can still be colored according to the number n of the element of the sequence that leaves a defined circle around the zero point first. The black set is called the "Mandelbrot set"; after the French mathematician Benoît B. Mandelbrot, who researched chaos and fractals. This is a deterministically chaotic set, since all calculations are based on fixed starting conditions and chance plays no role. The figure of this set, also called in German „Apfelmännchen“; appears at its edge as a bud in infinitely frequent similar repetitions.

Structures that consist of an infinite number of smaller copies of themselves are also called fractals, they show forms of self-similarity. Linear iteration functions do not lead to fractal structures.



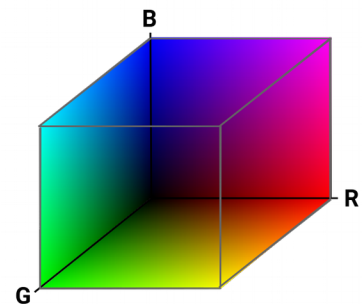
Apfelmännchen

Normally „benign“ functions are used for the iteration rule, which are complexly differentiable. In the example above this is the function $f(z) = z^2$. In mathematics, such functions are called holomorphic or analytic, they must meet certain conditions.

I have now created a program that allows me to use other functions for the iteration rule as well. For this purpose I specify the real and imaginary part of the function separately. Here, it is possible to use non-differentiable functions and even discontinuous functions. This makes it possible to create images that are even more „chaotic“ than the normal fractals.

How do I get to the pictures? First, I randomly try some simple iteration formulas. “Simple” means that only the four basic arithmetic operations occur, with which the real part **x** and the imaginary part **y** are linked. The image is then first calculated in the range for **x** and **y** from **-10** to **+10**. The numbers of iteration steps required are then displayed by default in the brilliant rainbow colors. I can then look where there are interesting areas and make an extract enlargement of them. The colors can then be changed by defining a possibly very crooked path through the **RGB** color space, even with mathematical functions.

If these images become too boring, you can still use trigonometric functions (sine, cosine, tangent etc.) also use the exponential function or the logarithm. It becomes really interesting if you use even more bizarre functions, such as the absolute value or the discontinuous jump function. This is zero for negative x and one for positive x. By putting these functions together, there is an indescribable amount of possibilities. You have to be lucky in your choice and guess the interesting areas. Sometimes special structures only appear at enormous magnification.



RGB-Farbenraum

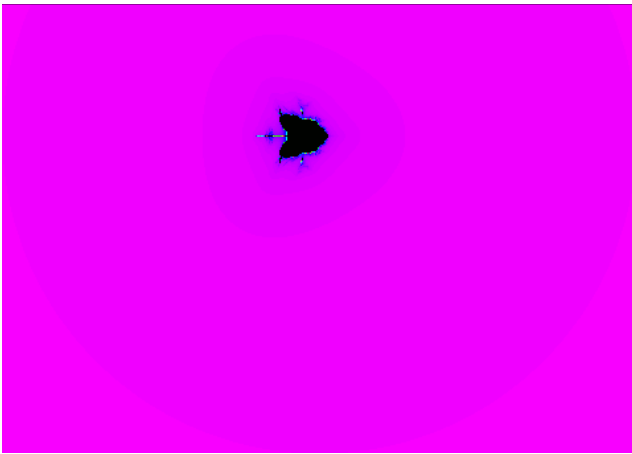
Here is an example of how a fractal image has been created in different steps:

The functions are chosen as iteration formulas:

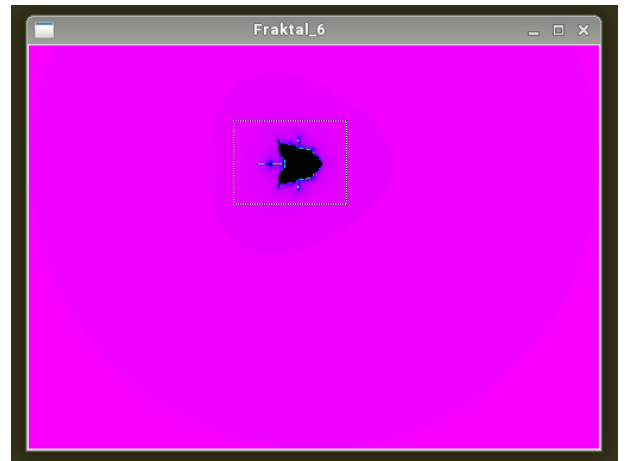
For the real part: $f(x,y) = x*x-y*y$
 For the imaginary part: $f(x,y) = 2*x*y+y$

In order to be able to determine whether the respective sequence remains in a fixed circle or not, the iteration number **200** is fixed. In this example, this means that if after **200** steps the circle with radius **100** is not yet exceeded, the sequence z_n is assumed to be limited. The corresponding starting points are colored black. The other points are colored differently depending on the number of steps in which the circle boundary is crossed for the first time. First in the form of rainbow colors on the outer edge of the **RGB** color space shown above.

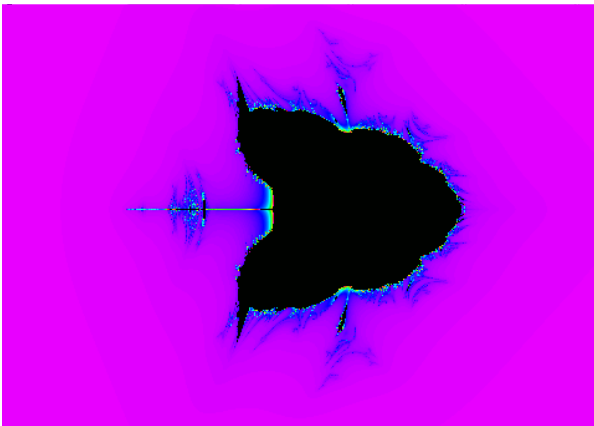
For the image width of **480** pixels and the image height of **340** pixels, the real section of variable **x** is selected from **-10** to **+10**, the imaginary section of variable **y** from **-10** to **+4.17**.



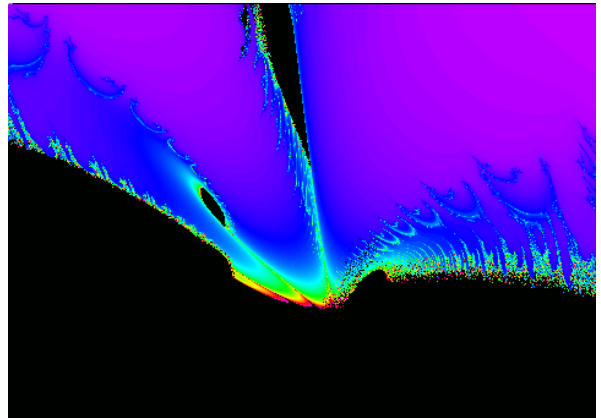
Stage 1: complete picture



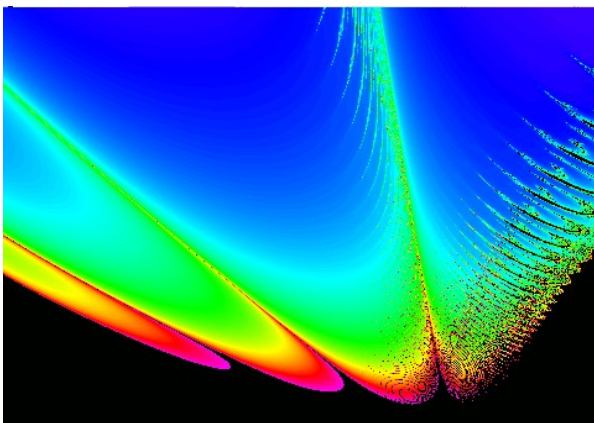
Stage 2: a section is selected



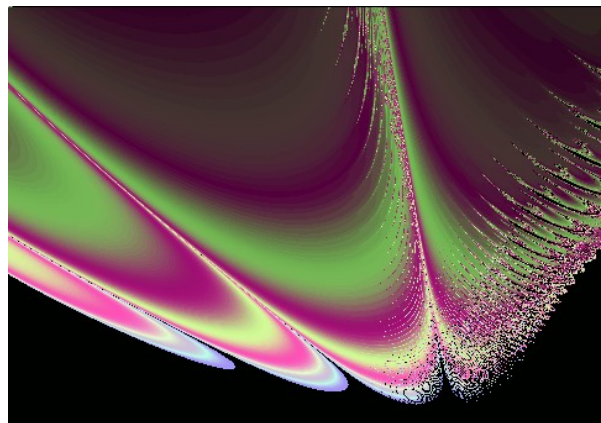
Stage 3: Cut-out



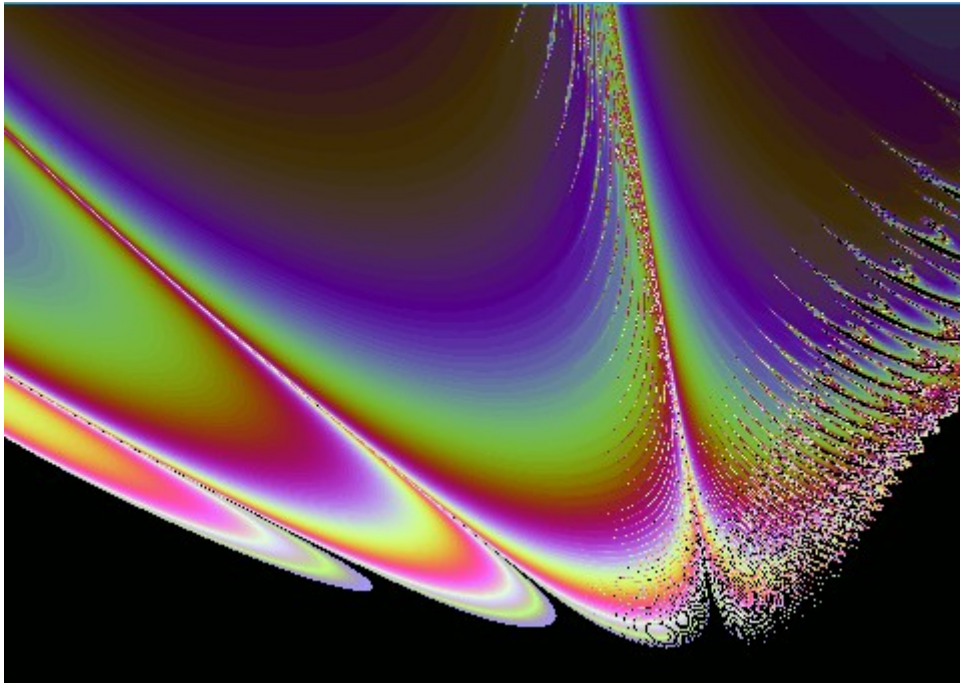
Stage 4: Smaller cut-out



Stage 5: Further enlargement



Stage 6: Other colours



Final stage: Final colour selection

For the final image, **100** times the resolution was calculated and saved. It was calculated according to the following iteration functions:

	Real part	imaginary part
Tubes in decay	$x*x-y*y$	$2*x*y+y$